CONTROLLING THE DEVELOPMENT OF INSTABILITY OF A CAPILLARY JET BY TWO ULTRASONIC BEAMS

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In fluid mechanics there has been growing interest lately in the transition from laminar to turbulent flows [1-6]. In experimental studies of the nucleation of turbulence and in the solution of practical problems of "controlling the laminar-turbulent transition" an important factor is the analysis of the mechanisms underlying the excitation of instability waves by various kinds of disturbances introduced into the flow. Growing as they move downstream, the induced waves can significantly alter the evolution of the spectrum of fluctuations, retarding or accelerating the development of turbulence. The excitation of Tollmien–Schlichting waves by a sound field in a boundary layer on a plate and their influence on the transition to turbulence have been investigated [1-4, 6]. The large disparity between the phase velocities of acoustic and hydrodynamic disturbances in subsonic flow causes the sound field to induce waves in the boundary layer only in the vicinity of the forward edge of the plate (localized excitation). The possibility of the generation of instability waves by two ultrasonic beams at resonance between the induced wave and the combination disturbance acting on the medium has been discussed [7]. In this case the wave excitation is distributed in the zone of intersection of the beams, and its frequency, which is equal to the difference between the frequencies of the ultrasonic waves, can be much lower than either.

Recent experiments with capillary jets [8, 9] can well be regarded as the first observations of the acoustic influencing of hydrodynamic flows. In these experiments an exceedingly high sensitivity of capillary jets to acoustic oscillations was observed, and it was noted that the action of the sound field on the jet is determined mainly by the vibrations of the nozzle [9]. The point at which the jet breaks up into droplets is easily ascertained visually from the characteristic thickening of the jet and loss of transparency of the flow. The excitation of instability waves under the combination influence of a sound field on the initial section of a jet is equivalent to increasing the level of the initial disturbances and will necessarily accelerate the droplet formation process.

1. Let us consider the combination mechanism described in [7] for the excitation of instability waves, in application to a capillary jet. Two sound waves with frequencies ω_1 and ω_2 are incident on a cylindrical liquid jet, in which they induce a difference-frequency wave $\Omega = \omega_2 - \omega_1$. We consider the excitation of an azimuthally symmetrical mode, which is responsible for instability of the jet. The excitation efficiency is determined by the deviation from resonance of the wave numbers of the nonlinearity coupled waves, which has the form

$$\Delta k = k_{2z} - k_{1z} - k_0 = k_0\left(\frac{\omega_1 v_0}{\Omega c_a}\sin\theta_1 + \frac{\omega_2 v_0}{\Omega c_a}\sin\theta_2 - 1\right), \tag{1.1}$$

where k_{1z} and k_{2z} are the projections of the incident wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ onto the z axis; $k_0 = \Omega/v_0$, real part of the wave number of the azimuthally symmetrical mode; v_0, flow velocity; c_a, sound velocity in air; $\theta_{1,2}$, angles between the x axis and $\mathbf{k}_{1,2}$ (the vectors $\mathbf{k}_{1,2}$ are situated in the plane of the x and z axes of a rectangular coordinate system). It follows from (1.1) that resonance of the interacting waves is possible only for $\Omega < (\omega_1 + \omega_2)v_0/c_a$. Inasmuch as $v_0 \ll c_a$, the frequencies $\omega_{1,2}$ must be much greater than Ω, which is in the flow instability band.

To determine the amplitude of the induced wave it is necessary to solve the system of equations of motion of the medium with proper boundary conditions at the disturbed interface. The problem is greatly simplified by the smallness of the density ratio of air to water $\rho_a/\rho_w \ll k$. An analysis shows that in the determination of the acoustic disturbance a not too thin (in comparison with the acoustic wavelength) jet can be treated as a perfectly rigid

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cylinder. In this case waves are excited in the jet only by the combination pressure exerted on its surface by the surrounding air.

To describe the sound field in tha atmosphere we use the system of equations of the adiabatic approximation [10]

$$\Phi_t + \frac{1}{2}(\nabla\Phi)^2 + \int\frac{dp}{\rho(p)} = 0; \qquad \rho_t + \mathrm{div}(\rho\nabla\Phi) = 0, \tag{1.2a}$$

in which Φ is the potential of the velocity field and ρ is the density of the medium. The boundary conditions are reducible to a zero net radial velocity component on the surface of the cylinder and the extinction condition at infinity. In the linear approximation the potential created by the scattering of each wave $\omega_{1,2}$ in the range of angles can be written in the form

$$\Phi = \frac{ip}{2\omega\rho_a}\sum_{m=-\infty}^{\infty}(-1)^m\left[I_m(\eta r) - \frac{I'_m(\eta a)}{H_m^{(2)\prime}(\eta a)}H_m^2(\eta r)\right]e^{i\omega t - ik_z z - im\varphi} + \text{complex conjugate},$$

where p is the complex pressure amplitude in the plane wave; r and φ are cylindrical coordinates ($x = r\cos\varphi$, $y = r\sin\varphi$); I_m and $H_m^{(2)}$ are Bessel and Hankel functions, respectively; a is the radius of the jet; and $\eta = k\cos\theta$ is the x component of the wave vector (we drop the indices 1 and 2 from p, ω, η, and k). The first part of the sum (1.3) represents the plane-wave potential expanded in cylinder functions, and the second part describes the sound field scattered by the cylinder.

Making use of the fact that the pressure fluctuations are small, we make the usual expansion of the function $\rho(p)$ in powers of $\tilde{p} = p - p(\rho_a)$: $\rho = \rho_a + \frac{1}{c_a^2}\tilde{p} + \cdots$. We seek the combination pressure, representing the solution of (1.2) by a perturbation series in the amplitude of the sound waves:

$$\Phi = \Phi^{(0)} + \Phi^{(1)} + \ldots, \quad \tilde{p} = p^{(0)} + p^{(1)} + \ldots, \tag{1.4}$$

where $\Phi^{(0)}$ is the sum of the potentials of the two incident waves, each with the form (1.3), $p^{(0)} = -\rho_a\Phi_t^{(0)}$ is the pressure corresponding to the potential $\Phi^{(0)}$. Determining the Fourier component $p^{(1)}$ and $\Phi^{(1)}$ with frequency $\omega_2 - \omega_1 \ll \omega_{1,2}$ and with wave number $k_{2z} - k_{1z} \sim k_0$, in the first-approximation equations we can neglect the term $\Phi_t^{(1)}$, which yields a contribution of the order* $v_0/c_a \ll 1$. Here the combination pressure inducing the wave in the jet is expressed in the form

$$p_\Omega^{(1)} = \frac{1}{2c_a\rho_a}[\tilde{p}^2]_\Omega - \frac{1}{2}\rho_a\left[\left(\frac{\partial\Phi^{(0)}}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\Phi^{(0)}}{\partial\varphi}\right)^2 + \left(\frac{\partial\Phi^{(0)}}{\partial z}\right)^2\right]_\Omega. \tag{1.5}$$

Here the index Ω refers to the amplitude of the Fourier harmonics: $p_\Omega = \langle p\exp(-i\Omega t)\rangle$, etc. (the angle brackets < > signify time averaging). Substituting (1.3) into (1.5), we obtain the following expression for the azimuthally symmetrical component $p_\Omega^{(1)}$ at r = a:

$$\bar{p}_\Omega^{(1)} = \frac{K}{\pi^2 a^2\eta_1\eta_2}\frac{\overset{*}{p}_1 p_2}{c_a^2\rho_a}\exp[-i(k_{2z} - k_{1z})z], \tag{1.6}$$

where the coefficient K is given by the relation

$$K = \sum_{m=-\infty}^{\infty}\frac{\left(1 - \frac{c_a^2 m^2}{\omega_1\omega_2 a^2} + \sin\theta_1\sin\theta_2\right)}{[H_m^{(2)\prime}(\eta_1 a)]^* H_m^{(2)\prime}(\eta_2 a)}.$$

We now inquire how expression (1.6) is changed by the incidence of two sound beams on the jet. We assume that the beam axes are oriented the same as the vectors $\mathbf{k}_{1,2}$ in the problem discussed above and that they intersect the jet axis at one point $z = z_0$ (we place the

*The estimate has been obtained for $k_z a$, $k_\perp a \sim 1$.

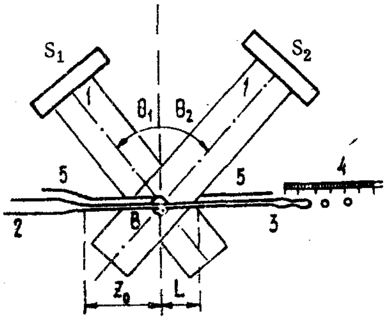

Fig. 1

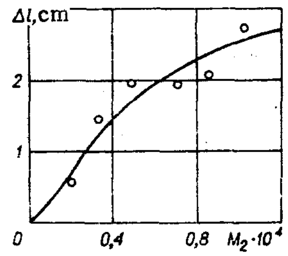

Fig. 2

origin at the start of the jet). Neglecting the variation of the pressure along the beams at distances $\sim L_* \sin\theta$ from the point z_0 (L_* is a characteristic length of the wave excitation zone), we represent the pressure amplitude in each of them in the form $p_{1,2} = p_{1,20}\chi(r_\perp)\times \exp[i\psi(r_\perp)]$, where $r_\perp$ is the distance to the beam axis, $p_{1,20}$ is the amplitude on the axis, χ is the normalized amplitude profile, and $\psi(r_\perp)$. When $\psi(r_\perp)$ and $\chi(r_\perp)$ vary only slightly in intervals $\Delta r_\perp \sim 2a\sin\theta$, the pressure created by the beam near the surface of the jet can be replaced by its value on the z axis (in the absence of the liquid). The spatial structure of the incident waves is essentially characterized in this case by replacing $p_{1,2}$ in (1.6) with $p_{1,2}(r_\perp)$ where $r_\perp = |z - z_0|\times\cos\theta_{1,2}$.

We confine the ensuing analysis to the combination excitation of long-wave jet disturbances, which canbe described within the context of the one-dimensional model [11]. The capillary pressure acting on the surface of the jet in our case must be added to the combination pressure determined above. The resulting system of equations is valid when the induced waves are large-scale ($k_0 a < 1$) and the same is true of the disturbances acting on the jet ($|k_{2z} - k_{1z}|a < 1$). The problem is reducible to the solution of a single equation for the amplitude of the azimuthally symmetrical component of the radial displacement ζ of the surface of the cylinder:

$$\left(i\Omega + v_0\frac{d}{dz}\right)^2\bar{\zeta}_\Omega + \frac{1}{2}\frac{\sigma}{a\rho\omega}\frac{d^2}{dz^2}\left(\bar{\zeta}_\Omega + a^2\frac{d^2}{dz^2}\bar{\zeta}_\Omega\right) = \frac{a}{2\rho\omega}\frac{d^2}{dz^2}\left[\bar{p}_\Omega^{(1)}e^{-i(k_{2z}-k_{1z})z}\right], \quad (1.7)$$

where σ is the coefficient of surface tension of water. We consider beams with a smooth profile ($\chi_z' \ll k_0\chi$, $\chi_{zz}'' \ll k_0^2\chi$) and limit the discussion to the case of small wave-number deviations ($|\Delta k| \ll k_0$). Then, putting $\bar{\zeta}_\Omega = (1/2)A(z)\exp(-ik_0 z)$ in (1.7), we obtain the following equation for the slowly varying complex wave amplitude A:

$$\frac{a^2 A}{dz^2} - \gamma^2 A = Q(z)\,e^{-i\Delta kz}, \quad (1.8)$$

where

$$Q = -\frac{(k_{2z} - k_{1z})^2 K p_1^* p_2}{\rho_a\rho_\omega c_a^2 v_0^2 \pi^2 a \eta_1\eta_2};\ \gamma = \frac{1}{v_0}\left[\frac{\sigma}{2\rho_\omega a}k_0^2\left(1 - k_0^2 a^2\right)\right]^{1/2}$$

is the spatial growth rate of the instability of the azimuthally symmetrical mode of the jet.

Equation (1.8) must be solved simultaneously with the boundary conditions at the start of the jet, which have the form $A(0) = A'(0) = 0$. The solution goes over asymptotically to a normal mode, which grows at the rate γ: $A \to A_0\exp(\gamma z)$, where A_0 is the initial amplitude of the induced wave. For small angles of incidence ($\cos\theta_{1,2} \approx 1$), such that the dependence of Q on the phase nonuniformity of the beams can be neglected, A_0 takes the form

$$A_0 = A_m\int_0^\infty \gamma\chi^2(|z - z_0|)\,e^{-\gamma z - i\Delta kz}dz. \quad (1.9)$$

Here $|A_m| = \frac{1}{2\pi^2}a\frac{\rho_a}{\rho_\omega}\left(\frac{c_a k_0}{v_0\gamma}\right)^2\left(1 + \frac{\Delta k}{k_0}\right)^2\frac{M_1 M_2|K|c_a^2}{\omega_1\omega_2\cos\theta_1\cos\theta_2}$ coincides with the amplitude of the induced wave when $\Delta k = 0$ and the beams are uniform; $M_{1,2} = p_{1,20}/c_a^2\rho_a$ denotes the acoustic Mach numbers at the point of intersection of the radiator axes with the jet axis.

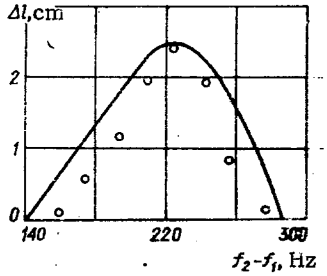

Fig. 3

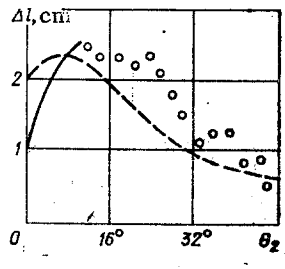

Fig. 4

As an example we find A_0 in the case where the beams are created by circular piston radiators embedded in rigid baffles and the point z_0 is located in the Fresnel zone at the same distance R_0 from the radiators. In accordance with the above-described experiments we set the Fresnel parameter $s = \lambda R_0/r_0^2$ equal to 0.74 (λ is the sound wavelength, and r_0 is the radius of the radiator). The pressure profile constructed for a given s in [12] can be approximated in the interval $0 < r_\perp < r_0$ by the function $\chi = \exp[-1.5\, r_\perp/r_0]/r_0]$, which is useful for the computation of the integral in (1.9). Assuming that $\chi \sim 1/r_\perp$ in the region $r_\perp > r_0$ we can estimate the error of the given approximation in an infinite z interval: $\delta A_0/A_m \approx 0.04\times \gamma a E_2(\gamma a)$ (E_2 is the power exponent of a second-order function). Finally, to ascertain the details of the combination action of the sound field on the jet we make the following substitution in (1.9):

$$\chi \to \chi(|z - z_0|)N(z - z_0), \tag{1.10}$$

where N(x) is the "baffle" factor (N = 1 for 0 < x < L, and N = 0 for x > L, x > 0). Then for $z_0 = 0$ we obtain the following relation from (1.9) for the amplitude of the excited wave:

$$\left|\frac{A_0}{A_m}\right| = \frac{\left|1 - e^{-\beta\gamma L - i\Delta k L}\right|}{\sqrt{\beta^2 + \left(\frac{\Delta k}{\gamma}\right)^2}} \left(\beta = 1 + \frac{3}{\gamma r_0}\right). \tag{1.11}$$

It follows from (1.10) that the amplitude of the excited wave scarcely increases when the length of the zone of action on the jet $L > L_* \approx 2/\beta\gamma$. The dependence on Δk on the right-hand side of (1.10) can be regarded as the resonance characteristic of the excitation, provided that the width of such resonance is sufficiently small. It is seen that in the transition to uniform beams ($r_0 \to \infty$) the length of the excitation zone increases, remaining finite ($L_* \to 2/\gamma$), while the width of the resonance decreases: $2\sqrt{3}\beta\gamma \to 2\sqrt{3}\gamma$.

2. In the natural state the jet breaks up into droplets due to random disturbances, which are transformed into an instability wave [9, 11]. The artificial excitation of a strong instability wave will necessarily accelerate the breakup of the jet. We now estimate the shift of the breakup point when acoustic irradiation is applied, proceeding from the following assumptions: 1) Random disturbances cause a random quasimonochromatic wave to be generated in the jet, with an amplitude that grows at the maximum rate* γ_m; 2) the interaction of the induced and random waves can be neglected; 3) both waves grow in accordance with the linear theory up to the formation of bottlenecks. The shift Δl of the breakup point toward the start of the jet is determined from the condition that the mean-square displacement of the surface of the jet is equal to its radius a:

$$|A_0|^2 e^{2\gamma(l_0 - \Delta l)} + a^2 e^{-2\gamma_m \Delta l} = a^2. \tag{2.1}$$

Here l_0 is the distance from the breakup point to the nozzle for $A_0 = 0$. When the condition $2(\gamma_m - \gamma)\Delta l \ll 1$ is satisfied, Eq. (2.1) is solved for Δl in explicit form:

$$\Delta l \approx \frac{1}{2\gamma_m} \ln\left(1 + \frac{|A_0|^2}{A_N^2}\right), \tag{2.2}$$

where $A_N = a \exp[-\gamma(\Omega) l_0]$ can be interpreted as the effective initial noise level.

3. For the experimental investigation of the combination influence of ultrasound on a jet we used the arrangement shown schematically in Fig. 1: S_1, S_2) pedestals with radiators

*The growth rate attains a maximum at the frequency $\Omega_m \approx (0.7/a)v_0$.

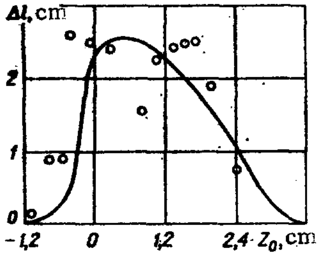

Fig. 5

(sound sources); 1) plates for mounting the bases of the pedestals; B) plate attachment screw; 2) nozzle; 3) jet; 4) measurement scale; 5) inserted baffle. The sources of ultrasound were circular lead barium titanate (PZT) ceramic wafers with a radius r_0 = 1.5 cm; they were mounted in pedestals, which were free to be rotated about a screw B situated at a distance R_0 = 9 cm from the sources. The working frequencies were chosen close to the principal resonance of the plates ($f_{1,2} = \omega_{1,2}/2\pi \approx 160$ kHz). The radiators could be displaced along the jet, thus varying the position of the point of intersection of their axes with the jet axis (distance z_0). The peak acoustic pressure on the axis of each radiator in air above the point B was measured with a microphone and attained 113 dB ($M_{1,2} \approx 10^{-4}$). The width of the amplitude-frequency response of the sound field of the radiators at the 0.7 level was 2 kHz. A jet of tap water issued at a velocity v_0 = 140 cm/sec from a nozzle of radius a = 0.075 cm. The natural breakup of the jet into droplets took place at a distance l_0 = 15.3 cm from the nozzle. The position of the breakup point was determined with a scale set up near the jet. The behavior of the jet was monitored visually in ordinary and strobe-light illumination. The latter was created by the application of a difference-frequency voltage obtained by mixing the voltages on the radiators.

For the above-indicated parameters of the system the instability growth rate has a maximum value γ_m = 1.03 cm^{-1} at a frequency $F_m = \Omega_m/2\pi \approx 210$ kHz. Owing to the low effective noise level, the shift of the breakup point according to expressions (1.9) and (2.2) is appreciable even for a comparatively low acoustic pressure. Taking the angle $\theta_1 \approx 12°$ for the purpose of estimation, we obtain the resonance value of the angle θ_2 = 5.5°. For small angles $\theta_{1,2}$ the calculations give $K \approx 8.2$. As a result, putting $M_{1,2} \approx 10^{-4}$, we obtain Δl = 2.4 cm.

The sound field was observed to have a significant influence on the breakup of the jet when the radiators were oriented toward the initial section of the jet and operated at close frequencies ($f_2 - f_1 \sim 200$ Hz). When two strong waves ($M_{1,2} \sim 10^{-4}$) were incident on the jet, the breakup point shifted toward the nozzle, and stabilization of the flow on the whole was observed (the spraying of droplets diminished). In strobed illumination we obtained a stationary pattern of droplets in the jet breakup zone, evincing their "combination origin." The breakup point shifted only when the higher-frequency (f_2) radiator was aimed downstream ($\theta_2 > 0$). In other words, to obtain an effective action on the jet the phase velocity of the combination field must have a component in the same direction as the liquid flow. The shift of the jet breakup point was measured for the following values of the fixed parameters: $M_{1,2} = 10^{-4}$, $\theta_{1,2} = 12°$, $f_2 - f_1 = 220$ Hz, z_0 = 0. The dependence of the shift Δl of the breakup point on the amplitude of one of the waves is shown in Fig. 2. No changes in the jet were observed when one of the sound waves was absent. Figure 3 shows the shift Δl as a function of the frequency difference between the incident waves. It is seen that the action is maximally effective when waves close to the maximum of the frequency dependence of the growth rate of the azimuthally symmetrical mode are excited in the jet. The action of the sound field was evident over a wide range of angles $\theta_{1,2}$ (Fig. 4) and persisted with a change in position of the point of intersection of the radiator axes with the jet axis (Fig. 5).

To exclude any possible influence of the combination pressure on the nozzle a thin baffle with a vertical slot was inserted as shown in Fig. 1. The opening of the slot from the point B in the flow direction was regulated between the limits 0 to 20 mm. In the presence of the baffle the interaction of the sound field with the jet was localized within the dimensions of the slot. The dependence of Δl on the width of the slot is shown in Fig. 6a. It is seen that the zone of wave excitation in the jet has a length of 6-7 mm (for values of L larger than these the shift essentially does not increase and is equal to the value without the baffle). Figure 6b shows the shift Δl as a function of z_0 for a slot width of 10 mm. The shift of the breakup point for large (in comparison with the sound wavelength λ = 2.1 mm)

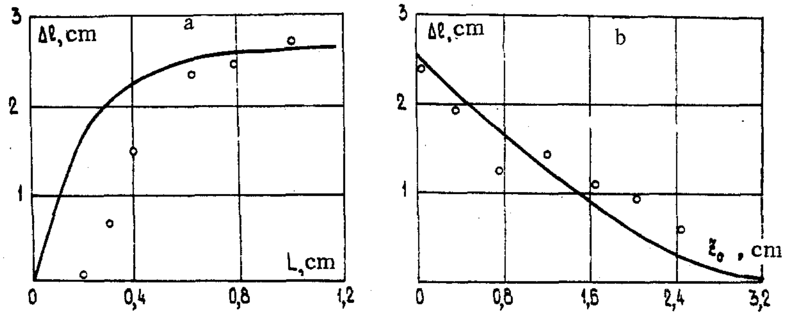

Fig. 6

distances of the point B from the nozzle eliminates oscillations of the tip of the pipet under the action of the sound field as a possible source of instability waves.

Simultaneously with the measurement results, Figs. 2-6 also show theoretical curves calculated according to expressions (1.9), (1.11), and (2.2). The theoretical dependence on the angle θ_2 in Fig. 4, which is plotted within the limits of validity of theory ($|\Delta k| \ll k_0$, $|k_{2z} - k_{1z}|a < 1$), encompasses only a small interval of angles and can be compared with the experimental points only for $\theta_2 \sim 12°$ (the overall dimensions of the pedestals are such that the source S_2 cannot be oriented at angles $\theta_2 < 12°$). The dashed curve in Fig. 4 represents the angular dependence obtained on the assumption of a narrow resonance peak with respect to Δk [it is assumed that the angular dependence of A_0 in (1.9) is governed by the factor $\exp(-i\Delta kz)$, whereas A_m is constant and equal to the value at resonance]. The calculations show that the resonance with respect to the angle in (1.9) is not narrow, because with a decrease of A_0 by one half it is impossible to neglect the variation of A_m. Because of the logarithmic behavior of the dependence of Δl on A_0, it is necessary to take into consideration the large deviations Δk even for a narrow resonance. To obtain the theoretical dependence on the angle for arbitrary θ_2 we must dispense with the simplifications used in calculating A_0. The characteristic width of the curve in Fig. 3 is determined mainly by the frequency dependence of the growth rate γ entering into the expression for A_N. We have carried out a theoretical calculation for the experiments with a slotted baffle, including the baffle factor in expression (1.9). The systematic shift of the experimental points to the right in Fig. 6a for a small slot width can be attributed to the presence of a finite gap between the baffle and the jet as well as to the finite width of the jet itself, which tends to shorten the zone of acoustic action on the jet with the radiators oriented at an angle with the normal.

Thus, in the above-described experiments with a capillary jet we have implemented a combination mechanism for controlling the development of instability by means of two ultrasonic beams. Regularization of the process of droplet formation and shifting of the breakup point toward the start of the jet were induced by the action of the sound field of two waves on the initial section of the jet, with localization of the action in the vicinity of the point of intersection of the radiator axes with the jet axis, where it resulted in the excitation of instability waves in the flow.

In conclusion we note some considerations of importance from the point of view of applications of the combination action of ultrasonic waves to the control of other hydrodynamic flows. Ultrasonic beams can be used to localize wave excitation in a predetermined flow zone, in which case the amplitude of the induced wave can be calculated with comparative simplicity in each specific situation. With the use of ultrasound it is possible to eliminate the disturbances created by the vibrations of flow-immersed plates and the working sections of pipes. Such a vibration background is unavoidable when the flow is irradiated with sound waves having a frequency equal to the hydrodynamic vibration frequency.

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BARO- AND THERMODIFFUSION OF A GAS MIXTURE IN A CAPILLARY

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UDC 532.529 + 532.72 + 533.6.001

In the presence of a solid surfaces confining the flow, the hydrodynamic and diffusional transfer of a gas mixture has a number of peculiarities which distinguish it clearly from the behavior of a mixture in free space. For example, in the analysis of slow flows of a mixture in a capillary, even in the region close to the viscous mode of flow (low Knudsen numbers), it proves important to allow for diffusional slippage at the channel wall, the contribution of the Knudsen boundary layers to the velocity components averaged over a cross section, etc. [1-4]. For this reason, in particular, expressions for the diffusional velocities of the components obtained within the framework of the ordinary kinetic analysis, and valid far from the walls [5, 6], prove to be not fully correct in a description of diffusional transfer inside a capillary.

Below we discuss the derivation of a general expression for the diffusional flux of a gas mixture in a capillary in the presence of longitudinal gradients of concentration, pressure, and temperature. This analysis is confined to the region of low Knudsen numbers ($Kn = \lambda/d \ll 1$, where λ is the effective mean free path of the molecules and d is characteristic transverse size of the channel). Under these conditions the averaged diffusional flux does not depend on the channel geometry in a first approximation with respect to the Knudsen number [4].

Let us consider the flow of a gas mixture in a channel bounded by two infinite parallel planes at $x = \pm d/2$. Let gradients of partial pressure and temperature exist in the z direction. For small $k_\alpha = p_\alpha^{-1} dp_\alpha/dz$ and $\tau = T^{-1}dT/dz$ the linearized kinetic equation for the mixture takes the form [7]

$$v_{\alpha z}\left[k_\alpha + \left(\beta_\alpha v_\alpha^2 - 5/2\right)\tau\right] + v_{\alpha x}\partial\Phi_\alpha/\partial x = \sum_\beta \hat{L}_{\alpha\beta}\Phi_\alpha, \tag{1}$$

where Φ_α is a nonequilibrium correction to the distribution function of particles of type α, defined by the equation

$$f_\alpha(\mathbf{v}_\alpha, x, z) = f_\alpha^{(0)}[1 + k_\alpha z + (\beta_\alpha v^2 - 5/2)\tau z + \Phi_\alpha(\mathbf{v}_\alpha, x)],\ f_\alpha^{(0)} = n_{\alpha 0}(\beta_\alpha/\pi)^{3/2}\exp(-\beta_\alpha v^2),\ \beta_\alpha = \frac{m_\alpha}{2kT_0}$$

(the index 0 corresponds to the parameters of an absolute Maxwellian distribution).

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